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NASA Contractor Report 156851

A Revised 5' Gravimetric Geoid and Associated Errors for the North Atlantic Calibration Area

**(NASA-CR-156851) A REVISED 5 MINUTE
GRAVIMETRIC GEOID AND ASSOCIATED ERRORS FOR
THE NORTH ATLANTIC CALIBRATION AREA Final
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Gerald L. Mader

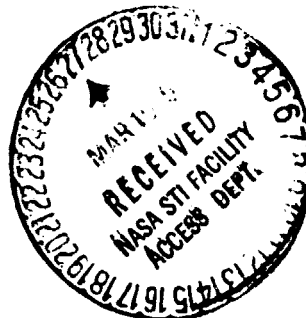
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CONTENTS

	<u>Page</u>
Introduction.	1
Fundamentals of Geoid Computation	2
Stoke's Integral	2
Satellite Height	4
Satellite Gravity.	6
$C_{2,0}^*$ and $C_{4,0}^*$	7
Surface Height	9
Gravity Data	11
Error Analysis.	13
Potential Coefficients	13
Gravity Anomalies.	15
Computation of the Revised 5' Geoid	17
References.	24

Appendix

The Unnormalized GEM-8 Potential Coefficients

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LIST OF FIGURES

<u>Figure No.</u>		<u>Page</u>
1	Satellite Height.	20
2	Surface Height.	21
3	Geoid Undulation.	22
4	Geoid Error	23

INTRODUCTION

Current detailed gravimetric geoids are obtained from a combination of observed free-air gravity anomalies and geoid undulations obtained from satellite derived potential coefficients. A number of such geoids have been computed on a 5' grid for the so-called North Atlantic Calibration area (Marsh and Vincent, 1973; Marsh and Chang, 1977). These geoids have been used in the same area to support GEOS-3 altimeter observations which are in turn used to monitor the Gulf Stream.

An examination of the calculations used in the derivation of these geoids as well as associated computer codes, has revealed a few minor errors in addition to some inconsistencies in the selection of fundamental constants. In view of this and with the availability of new 5' gravity data from the DMA gravity library in St. Louis, a revised 5' detailed gravimetric geoid for the calibration area has been derived. The method of calculation and the derivation of the constants used are explained in detail in this report. In addition, the errors of the geoid undulations, computed using the rms standard deviations of the gravity anomalies and estimates of the potential coefficient errors, are presented.

FUNDAMENTALS OF GEOID COMPUTATION

Stokes' Integral

A derivation of the Stokes' Integral yielding the geoid undulation, N , is presented in Heiskanen and Moritz (1967).

$$N = N_0 + \frac{R}{4\pi G} \iint \Delta g \ S(\Psi) \ d\sigma \quad (1)$$

where R = mean earth radius
 G = mean earth gravity
 Δg = surface gravity anomalies
 $S(\Psi)$ = Stokes' function
 $d\sigma$ = differential area

The Stokes' function is given by

$$S(\Psi) = \frac{1}{\sin(\Psi/2)} - 6 \sin(\Psi/2) + 1 - 5 \cos \Psi \\ - 3 \cos \Psi \ln \left[\sin(\Psi/2) + \sin^2(\Psi/2) \right]$$

$$\text{where } \Psi = \cos^{-1} \left[\sin \phi \sin \phi' + \cos \phi \cos \phi' \cos (\lambda - \lambda') \right]$$

and ϕ, λ is the geocentric latitude and longitude respectively of the computation point and ϕ', λ' refer to the position of the variable point of integration.

We also have

$$d\sigma = \cos \phi' \ d\phi' \ d\lambda'$$

The term, N_0 , contained in Eq. (1) is called the zero-order undulation and is given by

$$N_0 = \frac{k\Delta M}{2GR} - \frac{R\Delta g_0}{2G} \quad (2)$$

where

- k = Gravitational constant
- ΔM = difference between assumed and actual earth mass
- Δg_0 = global average gravity anomalies

If the actual mass and potential of the earth are not accurately known, Eq. (1) yields a geoidal surface consistent with the constants used but parallel everywhere to the actual geoid and offset by the constant amount N_0 . In general, because ΔM is not known and most geoid applications require only differences between geoid undulations, N_0 is set to zero.

The primary difficulty in performing the integration in Eq. (1) is that the desired gravity data is usually not available over the entire earth. For this reason the undulation is usually written as three components.

$$N = N_1 + N_2 + N_3$$

N_1 is called the satellite height. It is obtained from the potential coefficients and is equivalent to integrating the gravity anomalies, also obtained from the potential coefficients, over the entire earth. The second term, N_2 , is called the surface height and is obtained by integrating Eq. (1) over a finite area using the difference between the observed free-air gravity anomalies and those derived from the potential coefficients. The third term, N_3 would represent the extension of N_2 over the remainder of the globe, but since sufficient gravity data is lacking N_3 is usually set equal to zero. This latter action is justified since the decreasing weight of the Stokes' function with increasing Ψ will be applied to areas which, although increasing with Ψ , will tend to contain equal amounts of positive and negative gravity anomalies, thus yielding a negligible contribution. The computation of N_1 and N_2 is now discussed separately.

Satellite Height

In this section the derivation of the geoidal contribution that is a function of the potential coefficients, called the satellite height, is discussed. Also presented is the technique for obtaining the anomalous gravity field, called the satellite Δg , as a function of the potential coefficients.

A generalized form of Bruns' formula can be written

$$N = \frac{T - \Delta W}{\gamma} \quad (3)$$

where $\Delta W = W_0 - U_0$ the difference between the potential of the geoid and the potential of the reference ellipsoid. $T = W - U$ where W and U are the potentials at the computation point of the geoid and ellipsoid respectively. γ is the value of gravity at the computation point on the ellipsoid.

If M is the actual mass of the earth, we can write for the gravitational potential

$$W = \frac{kM}{r} \left[1 + \sum_{n=2}^{\infty} \left(\frac{a}{r} \right)^n \sum_{m=0}^n (C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda)) P_{nm}(\sin\phi) \right] + \frac{\omega^2 r^2}{2} \cos^2 \phi \quad (4)$$

where

- k = universal gravitational constant
- a = equatorial radius of reference ellipsoid
- r = geocentric radius of the point
- C_{nm}, S_{nm} = unnormalized spherical harmonic coefficients of the potential
- $P_{nm}(\sin\phi)$ = associated Legendre polynomials
- ω = earth's angular rotation rate

The potential at any point on a reference ellipsoid can be defined by

$$U = \frac{kM_0}{r} \left[1 + \left(\frac{a}{r}\right)^2 C_{2,0}(\text{ref}) + \left(\frac{a}{r}\right)^4 C_{4,0}(\text{ref}) \right] P_{nm}(\sin\phi) + \frac{\omega^2 r^2}{2} \cos^2\phi \quad (5)$$

where M_0 is the mass within the reference ellipsoid and $C_{2,0}(\text{ref})$ and $C_{4,0}(\text{ref})$ are reference values of the 2,0 and 4,0 terms of the spherical harmonic expansion. Traditionally, only these two terms have been used to define the reference ellipsoidal potential.

Substituting these two potentials into Bruns' formula yields

$$N = N_0 + \frac{kM}{r\gamma} \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=0}^n (C_{nm}^* \cos(m\lambda) + S_{nm}^* \sin(m\lambda)) P_{nm}(\sin\phi) \quad (6)$$

where we have assumed $M = M_0$ for the 2,0 and 4,0 terms and

$$N_0 = \frac{k(M-M_0)}{r\gamma} - \frac{(W_0 - U_0)}{\gamma} \quad (7)$$

where N_0 is again the zero order undulation and C_{nm}^* and S_{nm}^* denote the difference between observed and reference harmonic coefficients. In reality only $C_{2,0}$ and $C_{4,0}$ are differences, all other coefficients being equal to the observed values.

It is obvious from Eq. (7) that the value of N_0 will only contribute to a constant offset, hence the assumption that $M = M_0$ and $U_0 = W_0$ is often made and N_0 , if desired, can be added a posteriori. The satellite height can now be defined as

$$N_1 = \frac{kM}{r\gamma} \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=0}^n (C_{nm}^* \cos(m\lambda) + S_{nm}^* \sin(m\lambda)) P_{nm}(\sin\phi) \quad (8)$$

Satellite Gravity

The same generalized form of Bruns' formula will give the anomalous value of gravity between that observed on the geoid and the normal gravity on the reference ellipsoid, i.e.

$$\Delta g = -\frac{\delta T}{\delta r} - \frac{2T}{r} + \frac{2\Delta W}{r} \quad (9)$$

Using Eq. (3) and noting that $N = N_0 + N_1$ and substituting Eq. (7) yields

$$T = \gamma(N_0 + N_1) + \Delta W = \frac{k(M-M_0)}{r} + \gamma N_1 \quad (10)$$

Substituting this into the definition for Δg and performing the differentiation yields

$$\begin{aligned} \Delta g = & \frac{k(M-M_0)}{r^2} \\ & + \frac{kM}{r^2} \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=0}^n (C_{nm}^* \cos(m\lambda) + S_{nm}^* \sin(m\lambda)) P_{nm}(\sin\phi) \\ & + \frac{kM}{r} \sum_{n=2}^{\infty} n \left(\frac{a}{r}\right)^{n-1} \frac{a}{r^2} \sum_{m=0}^n (C_{nm}^* \cos(m\lambda) + S_{nm}^* \sin(m\lambda)) P_{nm}(\sin\phi) \\ & - 2 \frac{k(M-M_0)}{r^2} - \frac{2kM}{r^2} \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^n \sum_{m=0}^n (C_{nm}^* \cos(m\lambda) + S_{nm}^* \sin(m\lambda)) P_{nm}(\sin\phi) \\ & + \frac{2\Delta W}{r} \end{aligned} \quad (11)$$

which can be reduced to the more familiar form

$$\Delta g = \Delta g_0 + \frac{kM}{r^2} \sum_{n=2}^{\infty} (n-1) \left(\frac{a}{r}\right)^n \sum_{m=0}^n (C_{nm}^* \cos(m\lambda) + S_{nm}^* \sin(m\lambda)) P_{nm}(\sin\phi) \quad (12)$$

where, by analogy to N_0 , Δg_0 can be called the zero-order gravity anomaly

$$\Delta g_0 = \frac{-k(M-M_0)}{r^2} + \frac{2\Delta W}{r} \quad (13)$$

which for the same reasons is usually set to zero.

$C_{2,0}^*$ and $C_{4,0}^*$

The unnormalized reference values $C_{2,0}(\text{ref})$ and $C_{4,0}(\text{ref})$ are given by

$$\begin{aligned} C_{2,0}(\text{ref}) &= -J(n=2) \\ C_{4,0}(\text{ref}) &= -J(n=4) \end{aligned} \quad (14)$$

where J as a function of n is given by (Heiskanen and Moritz, 1967, p.73)

$$J_{2n} = (-1)^{n+1} \frac{3e^{2n}}{(2n+1)(2n+3)} \left(1 - n + 5n \frac{C-A}{RE^2} \right) \quad (15)$$

where

- C, A = moments of inertia
- R = principle radius of curvature
- E = $(a^2 - b^2)^{1/2}$
- a = semi-major axis
- b = semi-minor axis
- e = E/a

It is also found (ibid)

$$\frac{C-A}{RE^2} = \frac{1}{3} - \frac{2}{45} \frac{me'}{q_0} \quad (16)$$

where

- $m = \omega^2 a^2 b / kM$
- $e' = E/b$

and q_0 is given by q when $E/u = E/b = e'$ (ibid p. 71).

$$q_0 = \frac{1}{2} \left[\left(1 + \frac{3}{e'^2} \right) \tan^{-1}(e') - \frac{3}{e'} \right] \quad (17)$$

The past procedure has been to expand $\tan^{-1}(e')$ in a power series and develop expressions for $C_{2,0}(\text{ref})$ and $C_{4,0}(\text{ref})$ using the first two terms. There appears to be no reason to continue this because with present computer techniques e' , $\tan^{-1}(e')$, and hence q_0 can be determined to any desired accuracy. From the above definitions $C_{2,0}(\text{ref})$ and $C_{4,0}(\text{ref})$ can be found exactly and used to determine $C_{2,0}^*$ and $C_{4,0}^*$.

$$C_{2,0}^* = C_{2,0} - C_{2,0}(\text{ref})$$

$$C_{4,0}^* = C_{4,0} - C_{4,0}(\text{ref}) \quad (18)$$

Surface Height

If the potential field of the earth, as given in Eq. (4) were known to infinite degree and hence infinitesimal resolution, the geoid would be given exactly by Eq. (6). Unfortunately this series is known, with decreasing accuracy, only as far as degree and order 30. This yields a resolution or wavelength of 5° which is not sufficient for the detailed work that is desired. As a result, the computation of the geoid must be supported by surface observations of gravity anomalies.

The geoid undulations given by Eq. (8) are equivalent to those that would be obtained by integrating in Eq. (1) the satellite gravity anomalies given by Eq. (12). However, these satellite gravity anomalies are contained implicitly in the observed anomalies and to integrate the complete observed anomaly would mean accounting for the satellite field twice. Hence the satellite field is subtracted from the observed anomaly and only the residual is used in Stokes' integral. This geoid undulation, called the surface height is then added to the previous satellite height to yield the total geoid height. It should be remembered that the contributions to the observed gravity anomalies represented by the satellite anomaly is the only contribution that can be effectively integrated over the entire globe. The residual can be integrated only as far as the appropriate data exists.

The gravity residual then is given by

$$\Delta g' = \Delta g_{\text{obs}} - \Delta g_{\text{sat}} \quad (19)$$

where Δg_{sat} is given by the second term in Eq. (12).

For the purposes of computation the integral in Eq. (1) is recast as a double summation, where

$$N_2(\phi, \lambda) = \frac{R}{4\pi G} \sum_{i=-I}^I \sum_{j=-J}^J \Delta g'_{ij} S_{ij}(\Psi) \cos \phi'_i \Delta \phi'_i \Delta \lambda'_j \quad (20)$$

and $N_2 (\phi, \lambda)$ is the surface height at ϕ, λ , and $\Delta g'_{ij}$ is the residual anomaly and $S_{ij}(\psi)$ is the Stokes' function at the variable of integration position ϕ', λ' .

In computing 5' detailed geoids, N_2 is usually broken into 3 components

$$N_2 = N_a + N_b + N_c \quad (21)$$

where N_a is the primary contribution and represents the integration of 5' residual anomalies, in the present case, over a square area from 0° to 2° in each direction away from the computation point.

N_b includes the integration of residual anomalies on a 15' grid from 2° to 3° away from the computation point and N_c includes data on a 1° grid from 3° to 20° away from the computation point. To find the surface height at each 5' interval we add N_a to an average value of N_b and N_c using the values of N_b and N_c adjacent to N_a since, for the different resolutions, the 5' positions are not always coincident with the centers of the areas used to find N_b and N_c .

Gravity Data

Values for observed gravity anomalies have been obtained from the Defense Mapping Agency (DMA) gravity library at the Aerospace Center in St. Louis. They represent averages over 5' intervals of free-air point anomalies where the position of the average is at the center of the 5' cell and the cell is identified by the geodetic position of its northwest corner. Since the Detailed Geoid Computation (DGC) program uses the southwest corner as the cell identifier, the positions are corrected to the southwest corner to maintain consistency and avoid confusion.

The geodetic position of each cell is converted to geocentric coordinates and this position is used to compute satellite height, satellite Δg , and the residual gravity anomaly. These values are all assigned to the corresponding geodetic position. When the Stokes' integral is evaluated, the geodetic positions are again converted to geocentric coordinates to determine the subtended angle Ψ . The surface heights computed are also assigned to the geodetic position, and added to the satellite heights there to give a geoid undulation at the geodetic position.

The gravity anomaly is given by

$$\Delta g_{\text{obs}} = g_{\text{obs}} - \gamma$$

where the γ used by DMA is the value given by the Geodetic Reference System, 1967, International Association of Geodesy, Special Publications, No. 3, i.e.

$$\gamma = \gamma_e (1 + f_2 \sin^2 \phi + f_4 \sin^4 \phi) \quad (22)$$

with

$$\begin{aligned} \gamma_e &= 978031.85 \\ f_2 &= 0.005278895 \\ f_4 &= 0.000023462 \\ \phi &= \text{geodetic latitude} \end{aligned}$$

These quantities are computed from the adopted constants in the following way.

$$\gamma_e = \frac{kM}{ab} \left(1 - \frac{3}{2}m - \frac{3}{14} e'^2 m \right) \quad (23)$$

$$f_2 = -f + \frac{5}{2}m + \frac{1}{2} f^2 - \frac{26}{7}fm + \frac{15}{4}m^2 \quad (24)$$

$$f_4 = \frac{-1}{2} f^2 + \frac{5}{2}fm \quad (25)$$

These expressions are obtained from series approximations (Heiskanen and Moritz, 1967, p.76), but unlike the case for $C_{2,0}^*$ and $C_{4,0}^*$, these approximations can be tolerated. This is because all the expressions containing γ are simply scaled by its value, and minor differences will result in very small percentage errors. However, computing $C_{2,0}^*$ and $C_{4,0}^*$ requires the differencing of two nearly identical quantities that differ only in their 6th or 7th decimal places and hence are sensitive to truncating an infinite series. The results of the geoid calculations are also very sensitive to the values of $C_{2,0}^*$ and $C_{4,0}^*$ that are used and these numbers should be determined as accurately as possible.

The constants given in the Geodetic Reference System of 1967 and used by DMA to obtain gravity anomalies are not the same constants used in this report. To be consistent the following correction must be applied.

$$\Delta g_{\text{new}} = \Delta g_{\text{obs}} + \gamma_{1967} - \gamma_{\text{new}} \quad (26)$$

ERROR ANALYSIS

The accuracy of the geoid depends on the accuracies of the potential coefficients and the gravity data that are used in the geoid computation.

Potential Coefficients

The effect of potential coefficient errors on satellite height can be seen by examining one element of the double summation shown in Eq. (8).

$$N_1(n, m) = \frac{kM}{rY} \left(\frac{a}{r}\right)^n (C_{nm}^* \cos(m\lambda) + S_{nm}^* \sin(m\lambda)) P_{nm}(\sin \phi) \quad (27)$$

The differential change in $N_1(n, m)$ due to variations in C_{nm}^* and S_{nm}^* can be written

$$\Delta N_1(n, m) = \frac{kM}{rY} \left(\frac{a}{r}\right)^n P_{nm}(\sin \phi) (\Delta C_{nm}^* \cos(m\lambda) + \Delta S_{nm}^* \sin(m\lambda)) \quad (28)$$

If the errors in C^* and S^* are given in terms of the rms variations σ_c and σ_s we can obtain σ_{N_1} by squaring both sides of (28) and averaging over the number of values that are implied in σ_c and σ_s at the location n, m in the series.

$$\sigma_{N(n, m)}^2 = \left(\frac{kM}{rY}\right)^2 \left(\frac{a}{r}\right)^{2n} P_{nm}^2(\sin \phi) (\sigma_c^2(n, m) \cos^2(m\lambda) + \sigma_s^2(n, m) \sin^2(m\lambda)) \quad (29)$$

The cross term $\Delta C_{nm}^* \Delta S_{nm}^* \cos(m\lambda) \sin(m\lambda)$ has zero average since over many measurements of C and S at particular values of n and m , equal amounts of positive and negative products are likely, i.e., it is assumed that the errors in C and S are uncorrelated.

The total error to satellite height is found by adding the squares of all the individual errors within the summation.

$$\begin{aligned}
\sigma_{N_1}^2 &= \sum_{n=2}^{\infty} \sum_{m=0}^n \sigma_{N_1}^2(n, m) \\
&= \left(\frac{kM}{rY}\right)^2 \sum_{n=2}^{\infty} \left(\frac{a}{r}\right)^{2n} \sum_{m=0}^n (\sigma_c^2(n, m) \cos^2(m\lambda) + \\
&\quad \sigma_s^2(n, m) \sin^2(m\lambda)) P_{nm}^2(\sin\phi)
\end{aligned} \tag{30}$$

It is also necessary to determine the rms error in the satellite gravity, to compute the error in surface height arising from errors in the residual gravity. This is obtained in an exactly analogous manner to the satellite height errors and is given by

$$\begin{aligned}
\sigma_{\Delta g}^2 &= (kM/r^2)^2 \sum_{n=2}^{\infty} (n-1)^2 \left(\frac{a}{r}\right)^{2n} \sum_{m=0}^n (\sigma_c^2(n, m) \cos^2(m\lambda) + \\
&\quad \sigma_s^2(n, m) \sin^2(m\lambda)) P_{nm}^2(\sin\phi)
\end{aligned} \tag{31}$$

An additional error arises because the potential coefficients above $n=N_{\max}$ where in the present case N_{\max} is 30, are not known and cannot be included in the series. This so called error of omission has been estimated by Rapp (1973, 1975). Like the satellite height and gravity, this error is not a strong function of position and hence can be computed separately a posteriori and included with the other errors by the addition of small constant.

Computation of the errors given in Eq. (30) and Eq. (31) requires that $\sigma_c(n, m)$ and $\sigma_s(n, m)$ be known. According to Rapp (1975) the values of σ_c and σ_s are essentially independent of m and depend only on the degree n . He also shows that percentage errors in C and S increase approximately linearly with n . Thus, considerable effort may be spared by approximating

the potential coefficient errors as a linear function of degree rather than using the variance - covariance matrix of the harmonic coefficients.

Gravity Anomalies

The error that arises in the surface height due to errors in the gravity residuals can be found from Eq. (20) in an analogous way to the satellite height.

In this case, the contribution to $N_2(\phi, \lambda)$ from position ϕ', λ' is given by

$$N_2(\phi, \lambda)_{ij} = \frac{R}{4\pi G} \Delta g'_{ij} S(\psi)_{ij} \Delta \phi_i \Delta \lambda_j \quad (32)$$

Differential changes are given by

$$\Delta N_2(\phi, \lambda)_{ij} = \frac{R}{4\pi G} \Delta (\Delta g'_{ij}) S(\psi)_{ij} \Delta \phi_i \Delta \lambda_j \quad (33)$$

and the rms error at ϕ, λ due to Δg errors at ϕ', λ' is given by

$$\sigma_{N_2}^2(\phi, \lambda)_{ij} = \left(\frac{R}{4\pi G}\right)^2 \sigma_{\Delta g'_{ij}}^2 S(\psi)_{ij}^2 \Delta \phi_i^2 \Delta \lambda_j^2 \quad (34)$$

The errors at ϕ, λ from all ϕ', λ' that go into the integration are now

$$\sigma_{N_2}^2(\phi, \lambda) = \frac{R^2}{4\pi G^2} \sum_{i=-I}^I \sum_{j=-J}^J \sigma_{\Delta g'_{ij}}^2 S(\psi)_{ij}^2 (\Delta \phi_i \Delta \lambda_j)^2 \quad (35)$$

The gravity residual is given by Eq. (19) and its rms error is given by

$$\sigma_{\Delta g'}^2 = \sigma_{\Delta g_{obs}}^2 + \sigma_{\Delta g}^2 \quad (36)$$

where $\sigma_{\Delta g}^2$ is given by Eq. (31) and $\sigma_{\Delta g_{\text{obs}}}^2$ are the rms errors associated with the free air gravity anomalies.

COMPUTATION OF THE REVISED 5' GEOID

The constants adopted to compute the geoid described in this report are those associated with the GEM-8 potential coefficients (Wagner et al, 1977). The constants required to derive all the quantities used to compute the geoid are:

$$\begin{aligned}a &= 6378145. \text{ m} \\kM &= 3.986008 \cdot 10^{14} \text{ m}^3 \text{ sec}^{-2} \\f &= 1./298.255 \\W &= 7.2921151467 \cdot 10^{-5} \text{ sec}^{-1}\end{aligned}$$

and the values for mean earth radius and gravity are

$$\begin{aligned}R &= 6371000. \text{ m} \\G &= 979800. \text{ milligal}\end{aligned}$$

The semi-minor axis and the radius to any point on the reference ellipsoid are given by

$$\begin{aligned}b &= a (1-f) \\r &= a (1-f \sin^2 \phi)\end{aligned}$$

Using these constants, the reference gravity is given by

$$\begin{aligned}&= 978030.960508 (1 + 0.00527894364178 \sin^2 \phi \\&\quad + 0.0000232957478323 \sin^4 \phi)\end{aligned}$$

Since this formula differs slightly from that given by the Geodetic Reference System to which the DMA gravity data is referred, the following correction is added to the observed free-air gravity anomalies.

$$\Delta g'_{\text{obs}} = \Delta g_{\text{obs}} + 0.89 - .0428 \sin^2 \phi + .163 \sin^4 \phi$$

The unnormalized values of $C_{2,0}(\text{ref})$ and $C_{4,0}(\text{ref})$ are

$$C_{2,0}(\text{ref}) = -1.08264312976 \cdot 10^{-3}$$

$$C_{4,0}(\text{ref}) = 2.37097662187 \cdot 10^{-6}$$

Since the GEM-8 values are

$$C_{2,0} = -1.0826249 \cdot 10^{-3}$$

$$C_{4,0} = 1.6200262 \cdot 10^{-6}$$

$C_{2,0}^*$ and $C_{4,0}^*$ are given by

$$C_{2,0}^* = .18229758 \cdot 10^{-7}$$

$$C_{4,0}^* = -.75095042 \cdot 10^{-6}$$

A list of all the unnormalized GEM-8 coefficients that were used in these geoid calculations is given in Appendix I.

In order to compute the errors in the satellite height arising from potential coefficient errors, the following model for the coefficient errors was used.

$$\sigma_c, \sigma_s = 0 \quad N = 2, 3, 4$$

$$\sigma_c, \sigma_s = \frac{N-4}{16} \quad N > 4$$

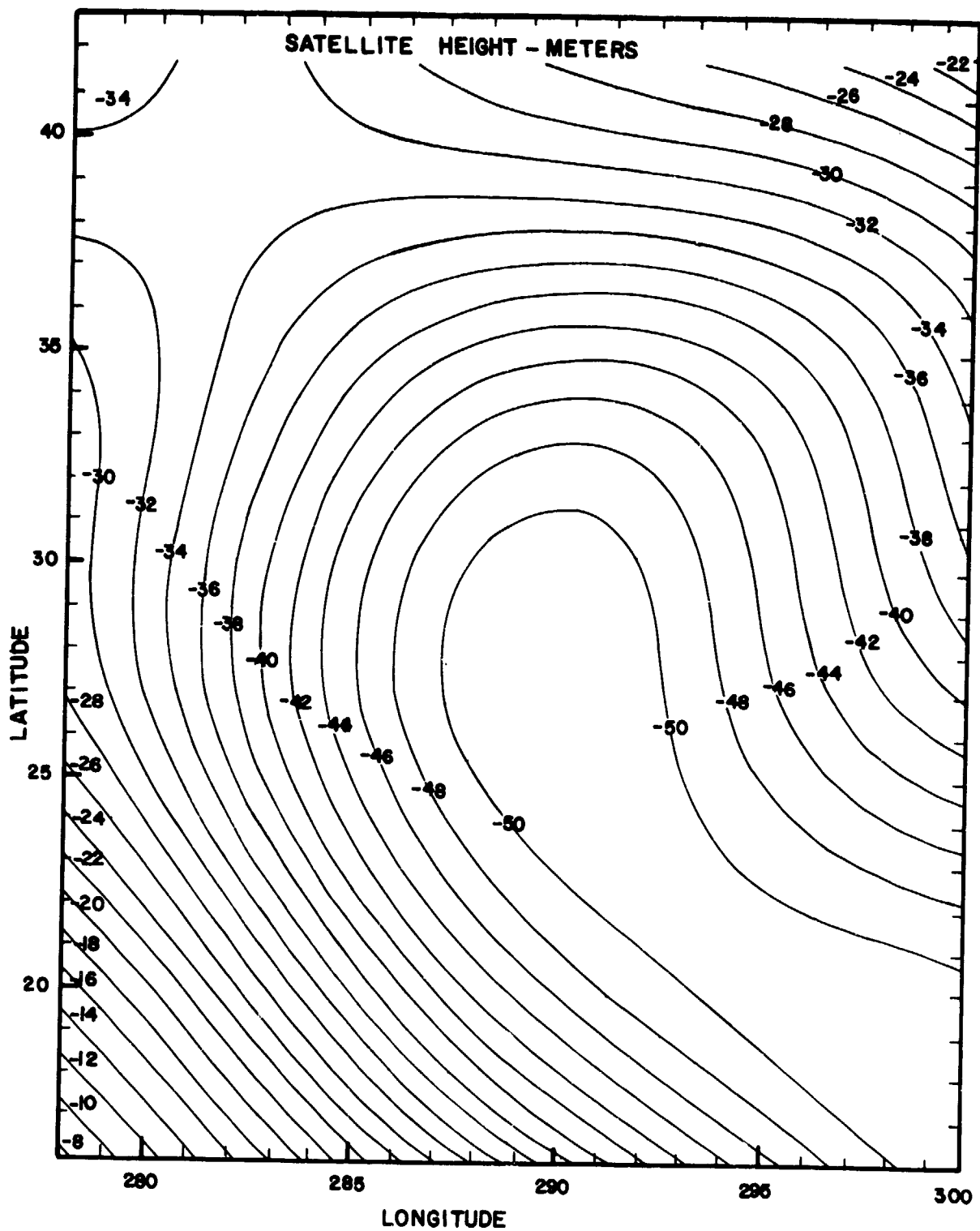
Hence, the lowest degree terms are most accurately known and the error increases linearly, reaching 100% at $N = 20$. This is an excellent representation of the coefficient errors given by Rapp and Rummel (1975) for GEM-6. Since the GEM-8 coefficients have been used, which should be an improvement over the GEM-6 coefficients, this error estimate may actually be a little conservative.

The errors in surface height were computed from the rms errors in the free-air gravity anomalies that accompanied the gravity data from DMA.

Using the constants listed in this section and Appendix I and following the method already described, a revised 5' gravimetric geoid was computed using the computer facilities of NASA/Wallops Flight Center. This geoid is the sum of the satellite and surface heights which are shown as contour maps in Figures 1 and 2, respectively. The geoid undulations are presented as a contour map in Figure 3. The contour interval for all these figures is 2 meters. The errors in the geoid have also been computed according to the method described and are shown as a contour map in Figure 4.

Acknowledgment: The cooperation of the Defense Mapping Agency Aerospace Center in St. Louis, Missouri, is gratefully acknowledged and appreciated.

Figure 1



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Figure 2

SURFACE HEIGHT - METERS

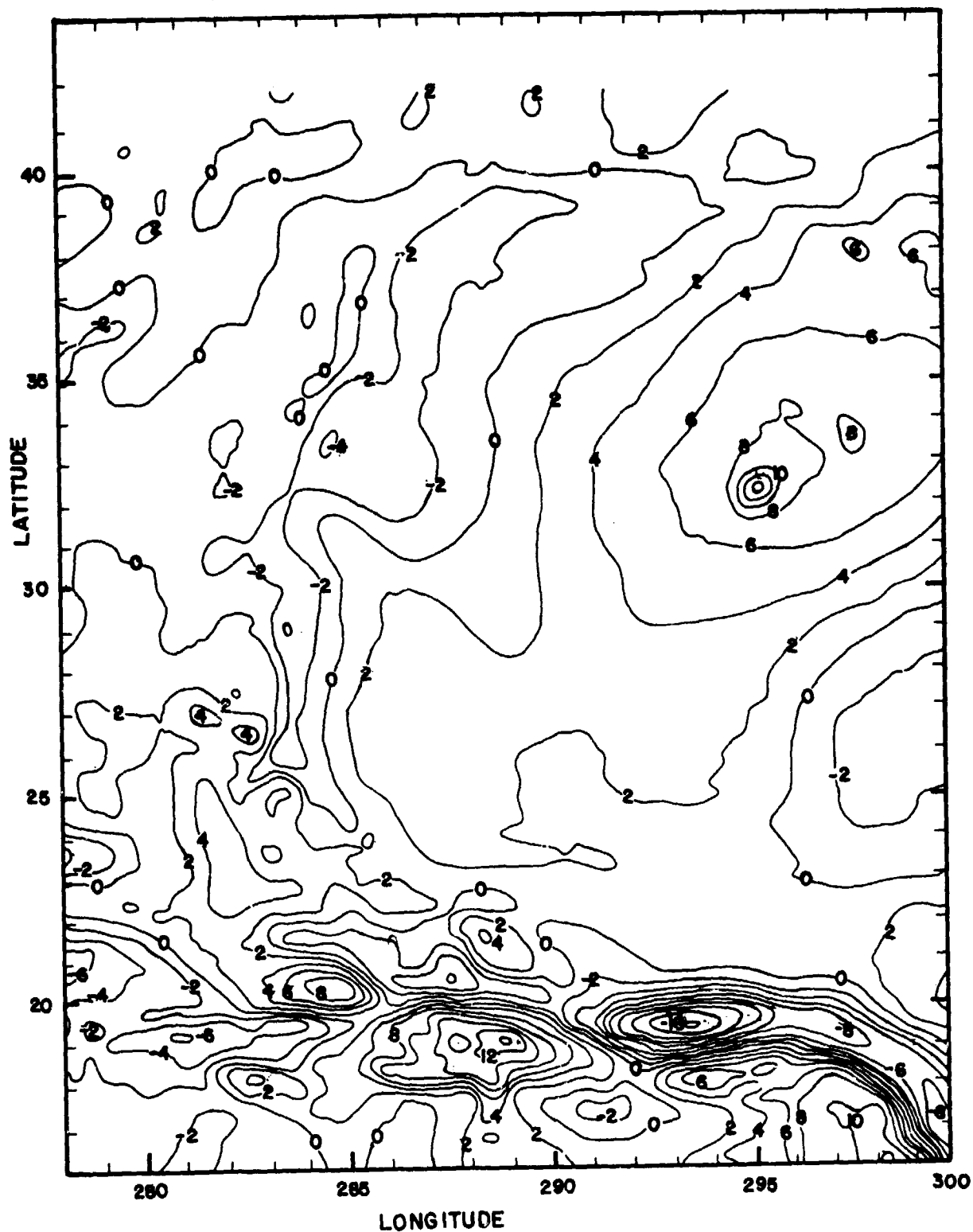


Figure 3

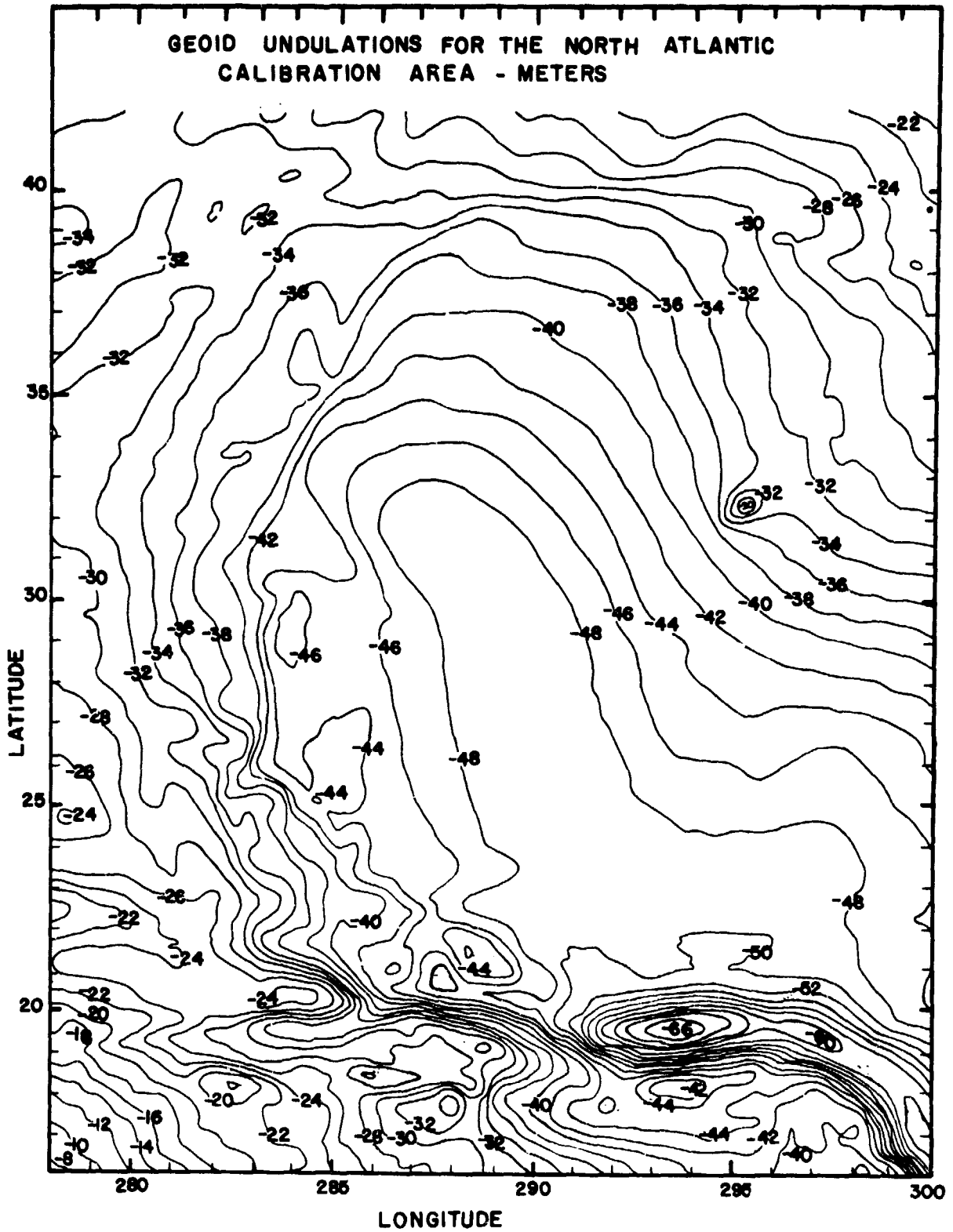
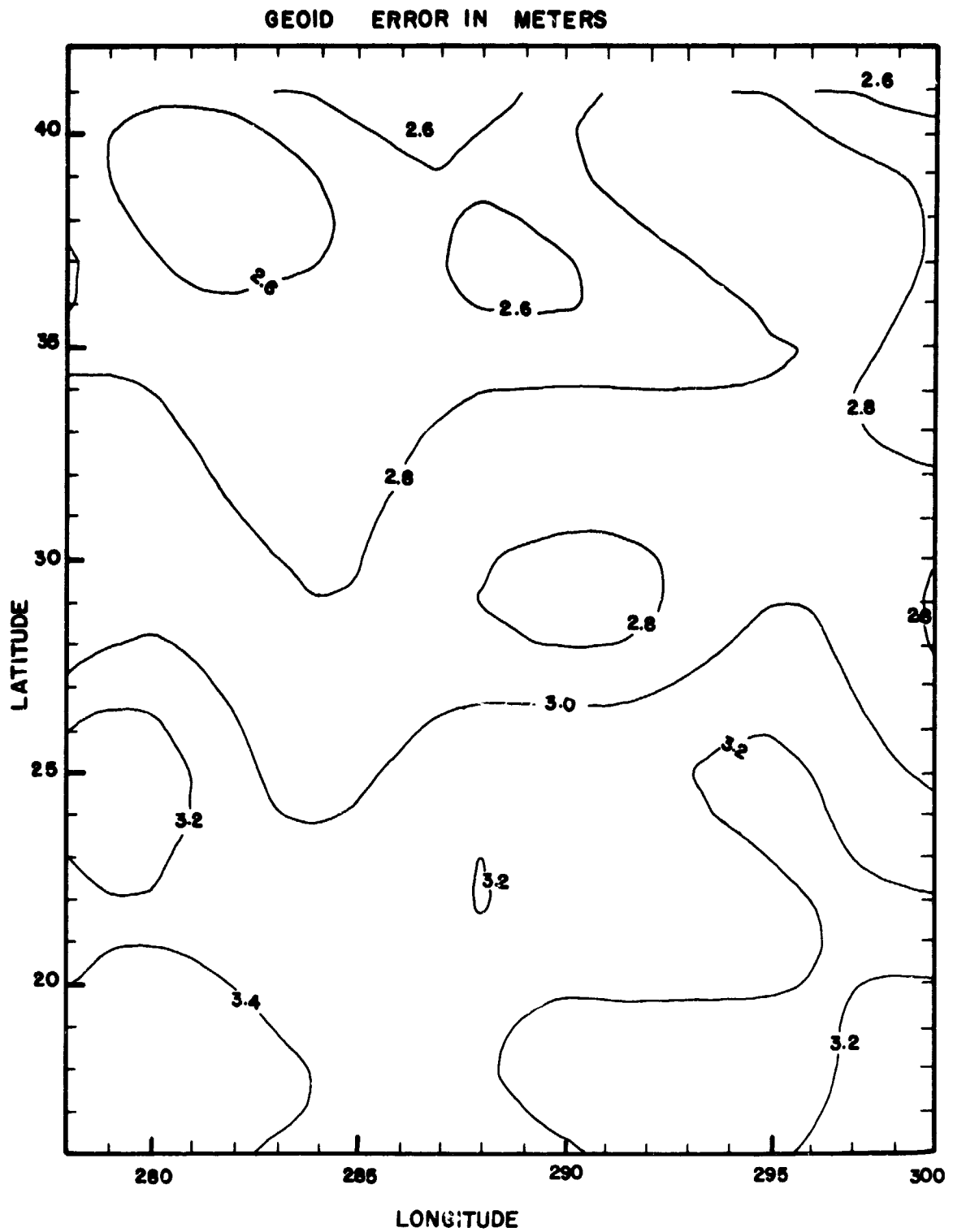


Figure 4



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APPENDIX

The Unnormalized GEM-8 Potential Coefficients

GEOD OF CONSTANT POTENTIAL REPRESENTED BY THE CONSTANTS OF GLF SYRM
UNNORMALIZED COEFFICIENTS

LOCALS									
INDEX	VALUE	INDEX	VALUE	INDEX	VALUE	INDEX	VALUE	INDEX	VALUE
2 0	5.39700E-08	3 0	2.35681E-06	4 0	-1.20000E-07	5 0	2.2582046E-07	6 0	-5.425556E-07
2 0	5.61220E-07	3 0	2.646031E-07	4 0	1.145574E-07	5 0	2.474613E-07	6 0	-2.12836E-07
12 0	1.66079E-07	13 0	1.891087E-07	14 0	-1.346697E-07	15 0	5.669597E-08	16 0	-1.758319E-08
12 0	6.311118E-08	18 0	4.917614E-08	19 0	8.703730E-09	20 0	1.137432E-07	21 0	3.365505E-08
22 0	5.676118E-09	23 0	-1.662912E-07	24 0	-1.607572E-08	25 0	-1.063337E-08	26 0	5.627112E-08
27 0	3.371357E-08	28 0	-3.326613E-08	29 0	2.299923E-08	30 0	0.		
SECTORIALS AND TESSERALS									
INDEX	VALUE	INDEX	VALUE	INDEX	VALUE	INDEX	VALUE	INDEX	VALUE
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3 1	2.194365E-07	4 1	2.497265E-07	5 1	3.067186E-07	6 1	-2.129443E-07	7 1	9.979910E-08
4 1	-5.096006E-07	5 1	-4.495087E-07	6 1	7.772351E-08	7 1	1.468450E-07	8 1	-1.175492E-08
4 4	-1.110907E-09	5 4	6.452270E-09	6 4	7.772351E-08	7 4	1.468450E-07	8 4	-1.175492E-08
5 1	-1.52278E-08	6 1	-7.191226E-08	7 1	1.070671E-07	8 1	-5.14207E-08	9 1	-6.751602E-09
5 4	-1.197611E-09	6 4	4.103165E-10	7 4	3.743105E-10	8 4	-1.678208E-09	9 4	-1.305092E-12
6 1	-1.09559E-08	7 1	2.582708E-08	8 1	6.533729E-09	9 1	4.530052E-08	10 1	-1.305092E-12
6 4	-1.09559E-08	7 4	2.582708E-08	8 4	6.533729E-09	9 4	4.530052E-08	10 4	-1.305092E-12
7 1	1.994435E-07	8 1	7.255414E-08	9 1	3.193775E-08	10 1	9.328772E-09	11 1	-3.068738E-09
7 4	-1.661944E-10	8 4	-2.648389E-10	9 4	-3.296255E-12	10 4	1.275903E-11	11 4	8.810648E-12
7 7	2.13336E-13	8 7	6.576351E-15	9 7	2.594933E-09	10 7	1.917429E-09	11 7	-4.733293E-09
8 1	1.351401E-08	9 1	2.856833E-08	10 1	6.101813E-09	11 1	5.847616E-09	12 1	-9.734458E-10
8 4	-1.04680E-10	9 4	8.806372E-11	10 4	-3.861152E-12	11 4	1.473766E-11	12 4	8.259595E-12
8 7	3.877970E-13	9 7	4.043637E-13	10 7	-1.381043E-13	11 7	1.437065E-13	12 7	-6.050093E-13
9 1	1.00021E-07	10 1	1.003801E-08	11 1	2.594933E-09	12 1	1.917429E-09	13 1	-4.733293E-09
9 4	-1.16632E-12	10 4	1.17703E-11	11 4	7.299864E-13	12 4	-5.61476E-12	13 4	2.537023E-12
9 7	-1.550862E-13	10 7	-1.191062E-13	11 7	7.646600E-14	12 7	-2.00096E-15	13 7	6.638666E-15
10 1	5.574633E-08	11 1	7.395095E-08	12 1	3.968581E-09	13 1	3.075124E-09	14 1	-8.08260E-10
10 4	-1.61780E-11	11 4	5.209978E-11	12 4	-3.768995E-12	13 4	-2.702949E-12	14 4	-6.611685E-13
10 7	1.440343E-14	11 7	3.51992E-15	12 7	5.13861E-15	13 7	-1.042273E-14	14 7	-8.64102E-16
10 10	4.299770E-16	11 10	9.897802E-17	12 10	-3.38261E-15	13 10	-1.042273E-14	14 10	-8.64102E-16
11 1	1.06440E-08	12 1	6.813359E-10	13 1	6.905268E-10	14 1	-5.055664E-09	15 1	-4.953370E-10
11 4	-1.92713E-11	12 4	3.688175E-11	13 4	1.370070E-12	14 4	4.045008E-12	15 4	6.361106E-14
11 7	5.51091E-15	12 7	3.73518E-14	13 7	4.891042E-16	14 7	3.080159E-15	15 7	4.098850E-16
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12 1	1.05788E-08	13 1	3.191700E-08	14 1	7.88979E-10	15 1	3.08099E-09	16 1	8.463280E-11
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12 10	-8.09970E-19	13 10	1.71440E-17	14 10	1.66666E-19	15 10	-1.912971E-19	16 10	-1.13273E-19
13 1	1.39281E-08	14 1	2.07991E-10	15 1	6.27091E-10	16 1	-1.344194E-09	17 1	1.869746E-10
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13 7	2.56003E-16	14 7	3.77030E-16	15 7	-2.84936E-17	16 7	1.62397E-16	17 7	7.58190E-17
13 10	2.81555E-18	14 10	-4.03908E-18	15 10	-4.40373E-19	16 10	3.26119E-20	17 10	1.66177E-19
13 13	-2.12801E-20	14 13	2.50090E-20	15 13	2.50090E-20	16 13	3.26119E-20	17 13	1.66177E-19

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